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NUMERICAL SIMULATION OF HYDRODYNAMICS OF A WEAKLY COMPRESSIBLE GAS-LIQUID MEDIUM

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A method for numerical simulation of the hydrodynamic parameters of a gas-liquid medium with allowance for its weak compressibility is proposed. Application of the method is illustrated by the example of the calculation of the hydrodynamics of a melt in a ladle during its filling and the blow.

Gas-liquid media can be encountered at all stages of steel production beginning with melting in converters upon their blow [1] and ending with out-of-furnace processing in ladles and casting where air penetration to the melt with the flow [2] or an inert gas being blown into a ladle at the stage of out-of-furnace processing [3] should be taken into account. A number of experimental investigations of the behavior of gas-liquid media under various conditions corresponding to the above-enumerated processes have been carried out in this context [4-6]; in addition, a series of mathematical models has been developed that make it possible to describe numerically the melt hydrodynamics with allowance for to the gas phase in both steel tapping (casting) [7] and the blow: within the Boussinesque approximation [8] and using more complicated model assumptions [9].

However, all known mathematical models including the above-enumerated ones do not account for an important property of the gas-melt medium, namely, its compressibility, which is a result of the considerable density of the melt. The high density of the melt results in relatively high pressure gradients in its volume. Thus, the ferrostatic pressure in a steel melt reaches atmospheric pressure at a depth of about 1.5 meters, whereas the melt level in the ladle can be as high as four or five meters. In this case a gas bubble floating from the bottom of the laddle up to the melt surface increases its dimensions approximately threefold. At the same time, air bubbles entrapped by the melt flow in the filling of ladles and ingot molds are compressed as they when get deeper into the melt, which decreases the volume gas content α of the melt and alters its hydrodynamic parameters. The dependence of α not only on the ferrostatic but also, most likely, on the dynamic pressure in the gas-melt medium can in certain cases affect its dynamics, for example, in studies of the action of the gas-liquid flow on the bottom of a ladle when it is filled.

From the preceding, both the physical models for water and the mathematical models being in reasonable agreement, mathematical models can produce incorrect results in the case of gas-melt media as a result of disregard for their compressibility.

In the present work a mathematical model is proposed for numerical investigation of the hydrodynamics of gas-liquid media at small gas content coefficients when the gas-liquid medium is weakly compressible. Here, as in [7], we assume continuity of the medium, the equations for which we write in a zero-order approximation in ρ_1/ρ_0 [10]:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\Delta\mathbf{V} + \mathbf{g}, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla(\rho\mathbf{V}) = 0. \quad (2)$$

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In this case the velocity of the medium V coincides with the velocity of the liquid phase; $\rho = (1 - \alpha)\rho_0$, and the inertial properties of the gas phase can be neglected, which allows us to use a diffusion approximation by complementing Eqs. (1) and (2) with the equation of mass conservation of the gas phase:

$$\frac{\partial \rho'}{\partial t} + \nabla (\rho' (\mathbf{V} + \mathbf{W})) = 0 , \quad (3)$$

where $\rho' = \alpha\rho_1$ is the density of the gas phase in the gas-liquid medium and the expression for the velocity of the gas phase with respect to the melt [11] $\mathbf{W} = -\mathbf{g}/gW$, where (in terms of our approximation $\rho_1/\rho_0 \to 0$)

$$W = \sqrt{\left(\frac{\sigma}{r_b \rho_0} + g r_b\right)} . \quad (4)$$

Expression (4) agrees well with experimental data on a single bubble floating up in both water [11] and a steel melt [12]. However, in order to apply this expression to the description of a gas-liquid medium one should determine experimentally the average radius of the gas-phase bubbles r_b, which depends on a number of factors, including the character of the flow. Therefore, in a number of cases, the value of W should be set based on various assumptions.

The assumption of the insignificance of α makes it possible to write Eqs. (1) and (2) within the Boussinesque approximation

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\,\mathbf{V} = -\,\nabla\tilde{p} + \nu_{ef}\Delta\mathbf{V} + (1 - \alpha)\,\mathbf{g} , \quad (5)$$

$$\nabla\mathbf{V} = 0 , \quad (6)$$

taking into account the presence of the gas phase solely in buoyancy forces. In the present work an algebraic model of turbulence [13] is used in which the effective coefficient of kinematic viscosity ν_{ef} is determined by the expression

$$\nu_{ef} = \nu + \frac{d}{\mathrm{Re}_d}\,V + (bd)^2 \left|\frac{\partial V}{\partial y}\right| , \quad (7)$$

which contains two parameters, d/Re_d and bd (when solving Eqs. (5) and (6) numerically, d is chosen to be equal to the subinterval of the spatial grid, and two parameters, the local, or grid Reynolds number Re_d, and b, the ratio of the mixing length to the grid subinterval, are used as turbulence parameters).

In deriving the equation for α we take into account that the gas present in the liquid obeys the state equation $\rho_1 = \rho(p)$ (dependences on the temperature and other factors are not taken into account) and the pressure p does not depend on time explicitly. Then we obtain from expression (3):

$$\frac{\partial \alpha}{\partial t} + \nabla (\alpha\,(\mathbf{V} + \mathbf{W})) = -\,\alpha\xi\,(p)\,(\mathbf{V} + \mathbf{W})\,\nabla p , \quad (8)$$

where $\xi = \partial \ln\rho_1/\partial p$. Thus, the transport equation for the coefficient of the volume gas content α includes the source, which is proportional to α, the scalar product of the gas phase velocity $\mathbf{V} + \mathbf{W}$ and the pressure gradient ∇p, and the state equation-dependent ξ factor, which equals zero within the approximation of the gas phase incompressibility ρ_1 = const. For a polytropic process

$$\rho_1 = \mathrm{const}\; p^{1/\gamma} \quad (9)$$

and $\xi = 1/\gamma p$, which will be assumed from this point on. At low pressure gradients, which take place when the liquid phase density is relatively small and flow is relatively steady, which is realized, e.g., in most gas-water filling and blow models under laboratory conditions, the source term in Eq. (8) can be neglected. In the case of gas-melt media, the pressure gradients are significant, as a rule, and the influence of the source term in (8) becomes nonnegligible.

Equations (5), (6), and (8), which have been supplemented with expressions for the effective coefficient of turbulent viscosity ν_{ef} and the gas phase velocity with respect to the liquid phase W, and the gas state equation, which can be chosen, e.g., in the form of (7), (4), and (9), form a complete system of equations for description of the dynamics of a weakly compressible gas-liquid system.

The presence of pressure in the source term for α is an argument in favor of employment of the natural velocity–pressure $\mathbf{V}-p$ variables in solving system (4)-(9). In this case we use the combined splitting method described in [7], which combines the advantages of the method of splitting over physical factors [14], for the hydrodynamic equations and the scaling difference scheme [15] for the equation of the convective gas concentration transfer. According to this method, the numerical solution of system (4)-(9) is performed in three stages:

$$\text{I)}\quad \tilde{\mathbf{V}} = \mathbf{V}^n + \tau\,[-(\mathbf{V}^n\nabla)\,\mathbf{V}^n + \nu_{ef}\Delta\mathbf{V}^n + (1-\alpha^n)\,\mathbf{g}\,]; \tag{10}$$

$$\tilde{\alpha} = \alpha^n - \tau\,(\mathbf{V}^n\nabla)\,\alpha^n; \tag{11}$$

$$\text{II)}\quad \Delta\tilde{p} = \nabla\tilde{\mathbf{V}}/\tau; \tag{12}$$

$$\text{III)}\quad \mathbf{V}^{n+1} = \tilde{\mathbf{V}} - \tau\nabla\tilde{p}; \tag{13}$$

$$\alpha^{n+1} = \alpha^n - \tau\,[(\mathbf{V}^n\nabla)\,\tilde{\alpha} + \tilde{\alpha}\,(\mathbf{V}^n + \mathbf{W})\,\nabla p^n/\gamma p^n\,], \tag{14}$$

where f^n is the value of the function f at time $n\tau$. The difference approximation of the right sides of expressions (10)-(14) is performed on a staggered grid by the conventional method [13].

The method proposed was used for calculations of the hydrodynamics of a melt in a ladle in two cases: with its filling and blowing with a inert gas via a slide gate. In both cases cylindric symmetry was assumed and cylindric coordinates were used. The programs for the calculations were coded in Pascal for an IBM PC. The turbulence parameters were taken as follows: $\mathrm{Re}_d = 2$, $b = 0.05$, and the polytropic exponent $\gamma = 1$, which corresponds to an isotherm.

In the problem of ladle filling, the boundary conditions for all quantities being calculated were set as in [7] and, as in [7], it was assumed that $W = 0$.

Preliminarily, in order to verify the adequacy of the model, a series of calculations of tank filling with water for regimes corresponding to those reported in [7] was carried out. In this case, as was assumed, results virtually coincident with the results of calculations presented in [7] were obtained. This, on one hand, substantiates the adequacy of the proposed method for description of the hydrodynamics of an air-water medium with a relatively steady flow (in the particular case, upon filling the tank), and, on the other hand, it is shown that compressibility of a gas-liquid medium does not manifest itself under these conditions, and simpler methods which do not account for compressibility can be used to describe its behavior.

Compressibility of the gas-liquid medium manifests itself if the ladle is filled with a steel melt. As a result of the high melt density (7000 kg/m^3) and significant penetration depth of the gas-liquid flow (several meters), its compressibility in this case affects the calculation results. Calculations were carried out for a 350-ton ladle with the following dimensions: mean radius R = 1.9 m and metal level H = 4.9 m. The effective radius of the flow was taken to be R_{fl} = 0.15 m, and the fraction of air injected by the flow α_0 = 0.7. The velocity of submerging of the flow for an almost completely filled ladle V_{fl} = 8 m/sec.

Results of the calculations for this case are presented in Fig. 1. In this figure, as in the ensuing one, the curves plotted with solid lines are related to the method proposed in the present paper, which accounts for the compressibility of the medium (version A). In order to reveal the role of compressibility of the medium the results

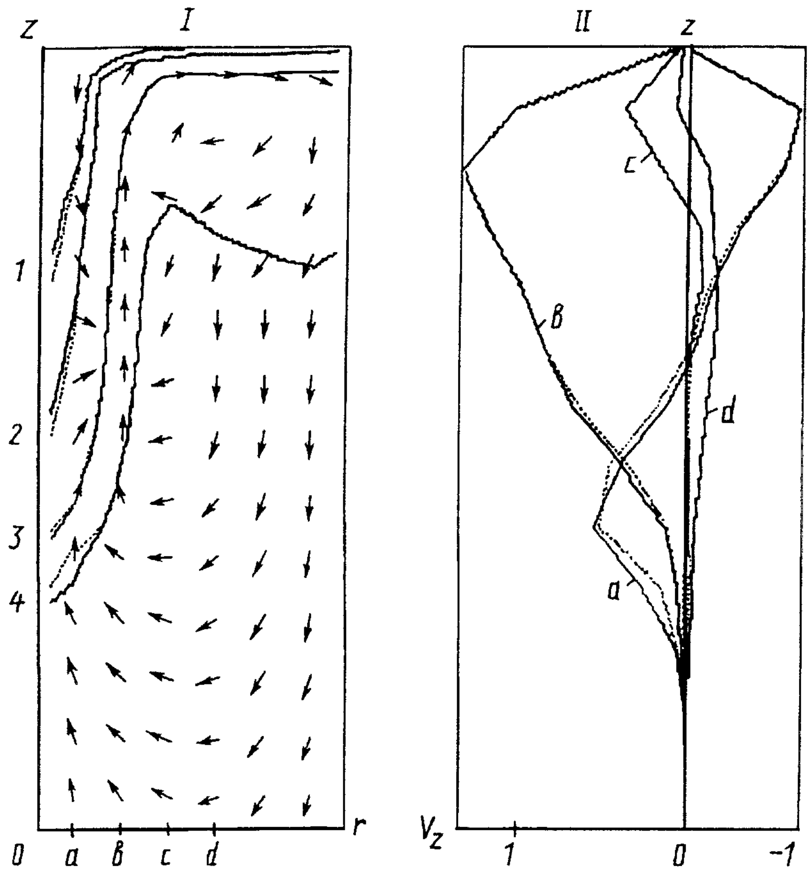

Fig. 1. Hydrodynamics of melt in ladle at the final filling stage. V_z, m/sec.

of calculations obtained under the assumption of its incompressibility are presented with dotted lines (version B). Figure 1, I presents the velocity and concentration fields, whereas Fig. 1, II plots the dependences of the vertical components of the melt velocities V_z on height z at the values of r marked in Fig. 1, I by letters. The corresponding dependences in Fig. 1, II are labelled with the same letters. The scale of the z axis in both halves of the figure is the same. It follows from calculations that the character of the melt motion in the ladle is, as a whole, much the same for versions A and B and the field of motion directions of the melt depicted in the figure by arrows virtually coincides for both of the versions. The difference in versions A and B starts to manifest itself in the field of the gas phase concentrations, which is presented in Fig. 1, I as a set of lines of equal gas concentrations corresponding to the following values: 1) 0.6; 2) 0.5; 3) 0.25; 4) 0.05. Whereas for relatively large values of the gas content coefficient α (curves 1 and 2), version A results in a faster decrease in α with depth, and curves 3 and 4, corresponding to large α, lie somewhat higher. This is due to the fact that at high α the compressibility of the gas-liquid medium is more pronounced than at small α. As a result, the total gas content in the flow is lower in case A, and the buoyancy force exerts a smaller braking action than in case B. This leads to deeper flow penetration in case A, which explains the lower disposition of the isolines of small α (when the medium is weakly compressible) for the version A. This is substantiated by Fig. 1, II, from which it follows that the maximum difference in the values of the melt velocities for versions A and B is observed at the depth corresponding approximately to the maximum penetration depth of the melt flow, and the closer to the symmetry axis, the greater the difference. The velocities for both versions virtually coincide with each other at distances from the ladle axis greater than half a radius. Attention is drawn to the coincidence of all curves (isolines of α and velocity plots) in the upper portion of the ladle. This suggests that the entire difference between the versions under consideration is due in this case to the ferrostatic pressure, which is substantiated by a control calculation with the dynamic pressure switched off when calculating the source term in the Eq. (8).

In the case of the blow problem, the boundary conditions on the inner ladle surface and on the free melt surface (on which the flow is now absent) are set in the same fashion as in the previous case. At the entry point

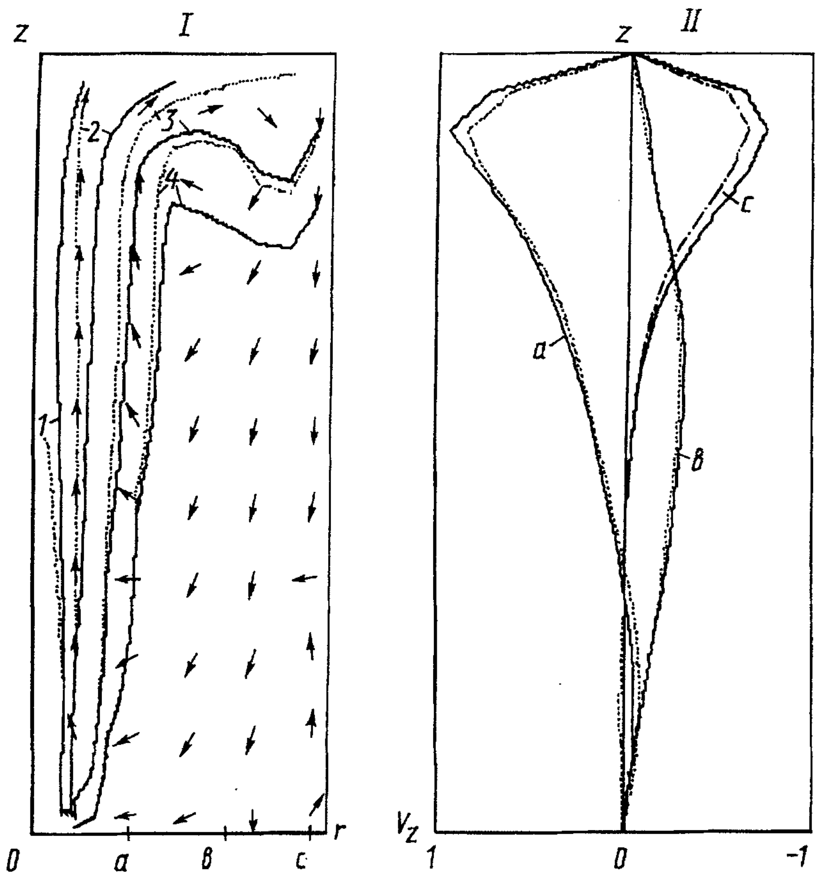

Fig. 2. Hydrodynamics of melt in ladle upon the blow.

of the gas flow at the ladle bottom, a zone of flow breakage can be selected outside which the flow is bubbling. The dimensions of this zone virtually do not affect the results of calculations for the rest of the ladle and at flow rates q of the gas close to 1 m^3/min, for which the results of calculations are presented in this work, it takes up several cells of the calculation grid. Calculations are performed within the entire volume of the ladle, including the selected zone, however, it is assumed that a gas-phase source acts within this zone whose intensity is determined by the quantity q. The velocity of the gas phase relative to the melt W in this case can be assumed to be equal with high confidence to 0.5 m/sec [6, 12].

Results of calculations of the ladle blow are presented in Fig. 2. Here the difference in versions A and B is even more pronounced. Whereas the general character of motion, which is determined by the field of velocity directions, is the same in both cases, as in the previous case, the fields of the gas phase concentration differ substantially. Figure 2, *I* presents the lines of equal concentrations for the following values of α: 1) 0.5; 2) 0.2; 3) 0.05; 4) 0.01. The entire difference between A and B can easily be explained by the expansion of bubbles upon floating up, which results in an increase in the buoyancy force acting on the gas flow, which leads to an additional increase in its velocity (Fig. 2, *II*). The greater difference of version A from B in the case of the blow compared with the case of filling is explained first of all by the fact that in this case bubbles overcome a greater pressure difference, since they move here from the bottom to the melt surface.

The results presented show that taking into account the compressibility of the gas-melt medium results in an improvement in the accuracy of determination of melt velocities by about 10% in the case of filling and blowing of the ladle. It is evident that the values of other parameters that affect substantially the results of the calculations such as, e.g., turbulence parameters or the gas phase velocity W with respect to the melt, should provide such accuracy. Otherwise the compressibility of the gas-melt medium can be neglected. However, as a result of the more substantial difference in the gas content coefficients in versions A and B, the consideration of compressibility can affect substantially other parameters, e.g., thermal characteristics.

It should be noted that the equation of state (9) adopted in the present work can be generalized. One can, for example, account for the possibility of the collapse of a gas bobble upon approaching the critical pressure. In this case the source term in Eq. (8) acquires an additional component proportional to the product of α and the substantive derivative of temperature. Refinement of the expression for the diffusion velocity of the gas phase W as a function of the character of the motion of the medium (the velocity of the medium and its derivatives, and turbulence parameters) also seems to be important. These questions appear to be important in solving certain problems of heat and mass transfer from the area of steel metallurgy, and we are going to consider them in the future.

CONCLUSIONS

1. The numerical method proposed allows us to describe adequately the motion of gas-liquid media of arbitrary density (including the cases when melts are used as a carrier liquid) at small coefficients of the volume gas content within the approximation $\rho_1/\rho_0 = 0$.

2. The method considered can be used, for example, in studies of the hydrodynamics of a melt in a ladle upon its filling and blowing. In this case it is sufficient to account for just the atmospheric and the ferrostatic pressures, whereas the dynamic pressure can be neglected.

NOTATION

V, V, velocity of the medium and its value; **W**, diffusion velocity of the gas phase; **g**, free fall acceleration; ρ, ρ_0, ρ_1, densities of the medium, the liquid, and the gas phases; p, pressure; $\tilde{p} = p/\rho$; α_0, α, μ, ν, σ, coefficients of the gas content in the flow, gas content in the medium, dynamic and kinematic viscosity, surface tension; ν_{ef}, effective dynamic viscosity coefficient; Re_d, b, its parameters: the grid Reynolds number and the ratio of the mixing length to the grid subinterval; γ, polytrope exponent; R, H, radius and height of the tank; R_{fl}, radius of the flow; τ, time step; d, grid subinterval.

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